SELF ADAPTIVE MESH SCHEME FOR THE FINITE ELEMENT ANALYSIS OF ANISOTROPIC MULTICONDUCTOR TRANSMISSION-LINES

Magdalena Salazar-Palma*, Félix Hernández-Gil**

* Departamento de Señales, Sistemas y Radiocomunicaciones
ETSI. Telecomunicación. Ciudad Universitaria. 28040 Madrid. SPAIN.
** Telefónica, Investigación y Desarrollo. Lérida, 43. 28020 Madrid. SPAIN.

ABSTRACT

A self adaptive mesh algorithm for transmission-lines Finite Element analysis is presented: it leads to an easy-to-use automatic FEM program in which the mesh corresponding to the domain discretization -necessary for the FEM application- is automatically well adapted to the structure under study, taking into account not only its geometry and materials, but field behavior and singularities. The method is based on the calculation of the error of the gradient conjugate solution of the structure FEM approach with a given coarse mesh. The error analysis gives information about the need of refining the grid, and which elements must be subdivided. Method application to the quasi-static approach of several anisotropic substrates microstrip-line structures is shown.

INTRODUCTION

In the past the Finite Element Method (FEM) has proved to be a powerful tool for microwave field problems analysis, difficult to solve by other methods (1),(2),(3).

Nevertheless, to achieve a good accuracy the FEM requires a mesh generation that must take into account not only structure geometry and materials, but field distribution and singularities. This implies a manual grid generation: a great difficulty to produce fully automatic accurate and efficient analysis programs.

This paper presents an adaptive mesh algorithm able to generate easy-to-use fully automatic analysis programs, solving the difficulties above mentioned. The proposed adaptive algorithm may be applied not only to a quasi-static approach but to any FEM formulation: discontinuities analysis, transmission-lines full-wave approach, etc.

Results of the method application to the quasi-static analysis of several multiconductor transmission-lines are shown: a good treatment of field singularities and line parameters excellent accuracy are obtained with moderate CPU time.

METHOD OF ANALYSIS

The cross section of a general anisotropic transmission-line structure is shown in figure 1.

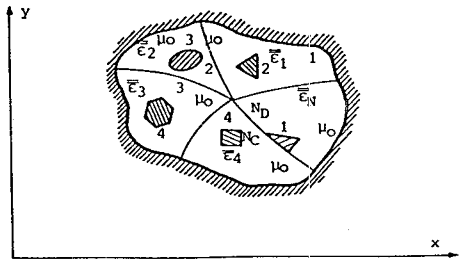


Figure 1. Cross section of a microwave transmission-line

Variational formulation of the quasi-static approach

The quasi-static approach of the electromagnetic problem of figure 1 structure, reduces to that of finding the scalar potential function $\phi(x,y)$ that minimizes the energy functional:

$$U = (1/2).\int_s (\bar{\nabla}\phi)^T.\bar{\varepsilon}.\bar{\nabla}\phi.ds \qquad (1)$$

written in matricial notation, where $\bar{\varepsilon}$ is the dielectric permittivity matrix (4),(5).

The corresponding Euler equation is the generalized harmonic one:

$$-\bar{\nabla}^T.\bar{\varepsilon}.\bar{\nabla}\phi = 0 \qquad (2)$$

Equations (1) and (2) are subjected to the appropriate Dirichlet or Neumann boundary conditions

For non-magnetic media, the N-multiconductor transmission-line parameters computation requires the U energy calculation of a set of different boundary conditions situations of the original problem, and a second set of the same problem, with all dielectrics removed (5),(6). Symmetry or antisymmetry allows to reduce the number of problems to be solved and the domain size under study.

Finite element discretization

For the FEM analysis each different dielectric subregion is discretized into a finite number of elements (7). A polynomial nodal approximation of the potential is employed in each of them:

$$\hat{\phi} = \sum_{i=1}^{M} N_i.\hat{\phi}_i = \bar{N}^T.\bar{\hat{\phi}} \qquad (3)$$

where $\hat{\phi}_i$ is the potential value at node i, N_i is the i finite element basis function of p order, being M the number of nodes of the element.

Substituting equation (3) into (1) and minimizing this functional, the equations for nodal values $\hat{\phi}_i$ derivation are obtained:

$$G(\hat{\phi},N_j)=\int_{s_e} \bar{\nabla}^T N_j.\bar{\varepsilon}.\bar{\nabla}\hat{\phi}.dS=\left(\int_{s_e} \bar{\nabla}^T N_j.\bar{\varepsilon}.\bar{\nabla}.\bar{N}^T.dS\right).\bar{\hat{\phi}}=0 \quad (4)$$

for each e element, being $j = 1,\ldots,M$.

The assembly of all elements equations and the introduction of the forced Dirichlet conditions lead to a global system:

$$\bar{K}.\bar{\hat{\phi}} = \bar{B} \quad (5)$$

that provides the desired potential nodal values $\hat{\phi}$ (7). A postprocess to compute the solution energy will give an upper bound $\hat{U}$ of the exact value U.

Finite element solution error analysis

In reference (8) it is demonstrated that the solution error -due to equation (4) approximation-satisfies the original boundary value problem with appropriate residuals substituting the equations second members. To measure the magnitude of the error the energy norm is selected; in two dimensions the error energy norm can be approximated by the global error estimate that will be the summation over all elements of the e element estimate:

$$\zeta_e^2=C.\left((h_e^2/\varepsilon_m.p).\int_{S_e} r^2.ds+(h_e/\varepsilon_m.p).\int_{\Gamma_e} J^2.d\Gamma\right) \quad (6)$$

where C is a constant that is independent of the e element, h_e is the largest side of the e element, ε_m is the permittivity matrix minimum eigenvalue, p is the polynomial degree, r is the e element surface residual

$$r_e = -\bar{\nabla}^T.\bar{\varepsilon}.\bar{\nabla}\hat{\phi} \quad (7)$$

and J, the jump of the $\bar{\varepsilon}.\bar{\nabla}\hat{\phi}$ vector normal component across the e element interfaces, Γ_e, with other elements or at the Neumann type boundaries.

FEM self adaptive algorithm description

The magnitude of the error decreases with an increasing number of nodes (7), but in any case it will be greater near sharp corners, due to field singularities (9),(10).

Thus the usual procedures to improve results accuracy are mesh subdivision and/or mesh grading that provide a greater density of nodes in regions with field singularities. The first procedure, when applied to uniform mesh, is not always efficient because it does not take into account field singularities. The second one implies a previous knowledge of field behavior.

A better procedure consists in an automatic self adaptive mesh generation, based in the error measurement of the FEM solution with a given discretization mesh. Thus if the FEM is applied to a coarse initial mesh, automatically generated from the structure description, several postprocesses of the solution will give the quasi-static energy U, the error local estimate and the error global estimate; a simple criterium for the adaptive process to take place is given by:

$$\sum_e \zeta_e^2 \le \lambda.U \quad (8)$$

if it is not achieved, the mesh will be automatically refined. The indication of which element is needed to be refined is simply obtained comparing each local estimate with the largest one, ζ_M^2. All elements having:

$$\zeta_e^2 \ge \gamma.\zeta_M^2 \quad (9)$$

will be refined, subdividing each of them into several elements. In order to achieve a new conforming mesh the refinement will be propagated, if necessary, to the contiguous elements, that will be also subdivided. To finalize the process two criteria may be used: that of equation (8), or a given maximum number of meshes.

IMPLEMENTATION OF THE FEM SELF ADAPTIVE SCHEME

The method above described has been applied, to the analysis of anisotropic, multidielectric and multiconductor transmission-lines structures, in order to obtain the quasi-static parameters.

A coarse initial mesh is automatically generated from the structure description: for thick and zero metal thickness the initial mesh generation algorithm is very simple; in the case of thin strips, it is a more elaborated one, based in the Delauny method, in order to provide non degenerated elements near the strip corners.

Linear and higher order ordinary triangular elements can be employed: the higher order ones are more efficient, give a better field description, and in their isoparametric version, provide the necesary tool for accurate curved boundaries consideration.

The assembled equations system is solved, for the initial mesh, with a direct Cholesky method, employing a sky-line matrix storage. For all other meshes only the non-zero coefficients are stored and a preconditioned conjugate gradient iterative method is employed; at each mesh the iterative process is initialized employing the previous mesh solution, interpolated, for the new nodes, by means of the basis functions of the previous mesh element to whom the new nodes belong: thus, a fast convergence to the equations system solution is achieved. Nodes reordering techniques are employed.

The λ and γ parameters of expressions (8) and (9) are selected by the user. For the element subdivision, the algorithm of reference (11) has been employed. This algorithm produces a sequence of triangulations with bounded angles, and with a smooth transition between small and large triangles. Both process finalizing criteria have been implemented.

RESULTS

The self adaptive algorithm was checked applying it to structures with analytical solution and studied by other authors. Figures 2a)-f) correspond to a square coaxial structure: the mesh evolution shows the expected corner refinement due to field singularities. Table I compares the results for impedance and capacitance with previous ones, showing an accuracy improvement of the corresponding bounds. CPU time for same order of accuracy can be compared with results of FEM analysis over uniform meshes of increasing density shown in Table II: CPU time refers to the whole process of uniform mesh refining, until the mentioned mesh is reached. Figures 3a)-c) show a symmetric stripline structure of zero metal thickness, the initial mesh, and one mesh step, showing the refinement due to the strong singularity of the zero metal thickness strip corner. Table III compares results with previous ones.

Figures 4a)-d) correspond to a single microstrip-line structure: results (table IV) are in agreement with those calculated by the method proposed in (14) and show the interest of metal thickness consideration if good accuracy is desired.

In figures 5a)-c) it can be seen an asymmetric coupled suspended lines structure over an anisotropic substrate -sapphire with non girated axes-, a mesh step of the configuration corresponding to the C_{22} parameter calculation and the equipotential lines, that show how the mesh is well adapted. Results are shown in table V.

The example of figures 6a)-b) corresponds also to an asymmetric suspended coupler with the strips in different planes and with anisotropic substrate. Results are shown in table VI.

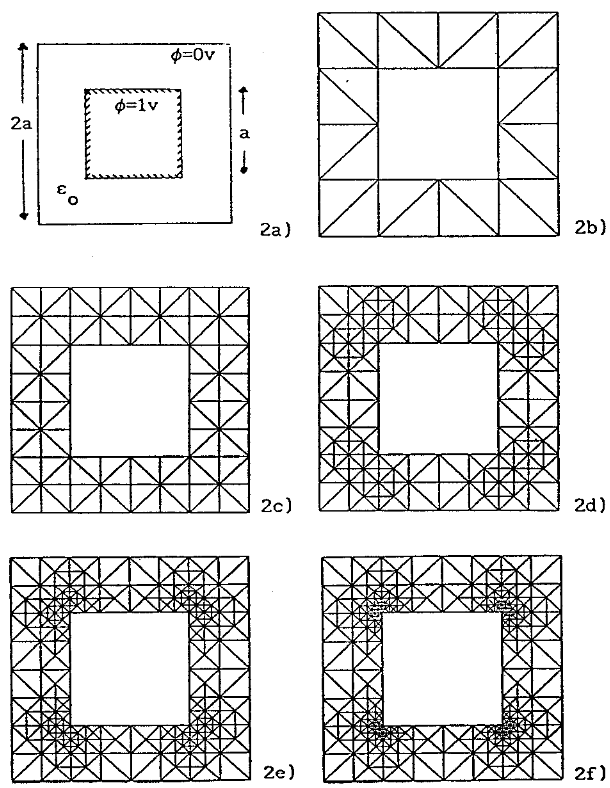


Figure 2. Square coaxial structure
a) Structure b)-f) Mesh evolution

This method with	$Z_o(\Omega)$	C_o/ε_o	Error (%)	CPU time VAX 8800
4 mesh steps	36.75400	10.250048	0.155	13 sc
5 mesh steps	36.78606	10.241116	0.068	18 sc
6 mesh steps	36.79878	10.237576	0.033	23 sc
7 mesh steps	36.80681	10.235342	0.012	28 sc
8 mesh steps	36.80916	10.234687	0.005	35 sc
Exact (12), (13)	36.81132	10.2341		
Reference (12)*		10.3644	*Upper bound	
Reference (13)**	36.7921		**Lower bound	

Table I. Square coaxial structure, p=2

Number elements	Number nodes	$Z_o(\Omega)$	C_o/ε_o	CPU time VAX 8800
3	12	36.01883	10.459259	3 sc
12	35	36.48192	10.326493	6 sc
48	117	36.67962	10.270883	10 sc
192	425	36.75896	10.248663	16 sc
768	1617	36.79053	10.239872	35 sc
3072	6305	36.80306	10.236386	166 sc

Table II. Square coaxial structure, p=2, uniform mesh

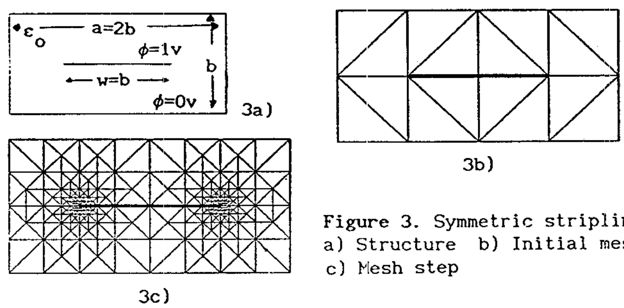


Figure 3. Symmetric stripline
a) Structure b) Initial mesh
c) Mesh step

This method with	$C_o/(4\ \varepsilon_o)$	Error (%)	CPU time VAX 8800
4 mesh steps	1.483990	0.999	14 sc
5 mesh steps	1.476670	0.745	19 sc
6 mesh steps	1.473025	0.380	25 sc
7 mesh steps	1.471206	0.135	31 sc
8 mesh steps	1.470298	0.073	39 sc
9 mesh steps	1.469822	0.060	44 sc
Exact (12),	1.46922		
Reference (12)*	1.51899	*Upper bound	

Table III. Symmetric stripline, p=2

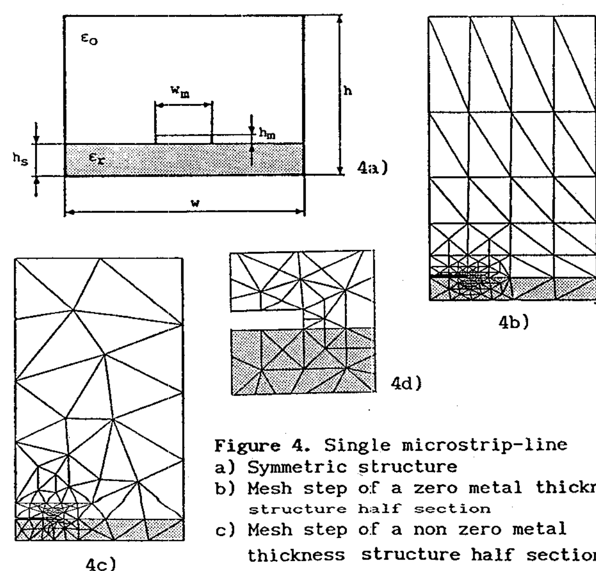
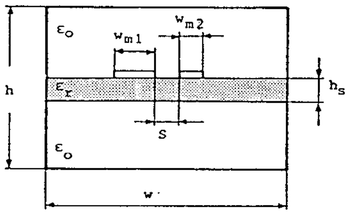


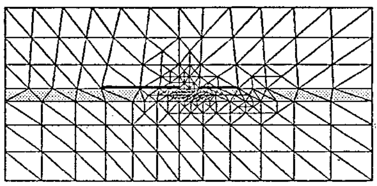
Figure 4. Single microstrip-line
a) Symmetric structure
b) Mesh step of a zero metal thickness structure half section
c) Mesh step of a non zero metal thickness structure half section
d) Zoom over strip corner of figure 4c)

	Z_o (Ω)	ε_{ef}	v_f 10^8 m/sc
h_m=0.0mm	48.93481	1.845903	2.206562
h_m=0.018mm	48.01976	1.832075	2.214874

Table IV. Single microstrip-line parameters
w=3.56mm, h=3.272mm,
w_m=0.8mm, h_s=0.254mm, ε_r=2.17, p=2



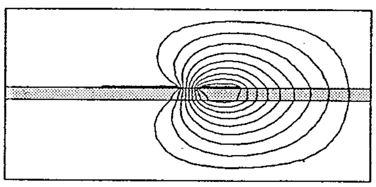
5a)



5b)

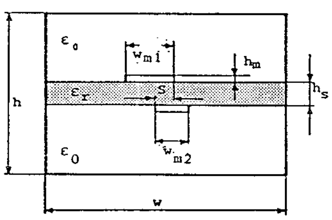
C_{11} = 0.805397.10^{-10}
C_{22} = 0.696536.10^{-10}
C_{12} =-0.419785.10^{-10}
C_{011}= 0.337606.10^{-10}
C_{022}= 0.258260.10^{-10}
C_{012}=-0.086791.10^{-10}

Table V. Capacitance matrix coef. (F/m)
w=7.12mm, h=3.56mm,
h_s=0.254mm, h_m=0.0mm,
w_{m1}=1.5mm, w_{m2}=0.75mm
s=0.4mm, p=2, ε_{11}=9.4,
ε_{12}=0, ε_{22}=11.6

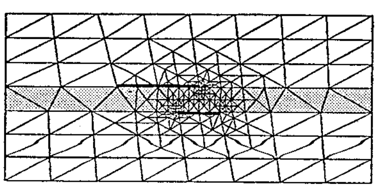


5c)

Figure 5. Asymmetric coupled suspended lines
a) Structure b) Mesh step for ϕ_1=0v, ϕ_2=1v
c) Equipotentials lines for ϕ_1=0v, ϕ_2=1v.



6a)



6b)

C_{11} = 0.211618.10^{-9}
C_{22} = 0.192196.10^{-9}
C_{12} =-0.164695.10^{-9}
C_{011}= 0.419336.10^{-10}
C_{022}= 0.316218.10^{-10}
C_{012}=-0.172641.10^{-10}

Table VI. Capacitance matrix coef. (F/m)
w=7.12mm, h=3.56mm,
h_s=.508mm, h_m=0.mm,
w_{m1}=1.6mm, w_{m2}=.8mm,
s=.4mm, p=2, ε_{11}=9.4,
ε_{12}=0, ε_{22}=11.6

Figure 6. Suspended offset parallel coupler
a) Structure b) Mesh step for boundary conditions ϕ_1=0v, ϕ_2=1v.

CONCLUSIONS

A self adaptive algorithm for FEM analysis of quasi-TEM structures has been presented. It provides a new method for mesh generation leading to automatic FEM programs with an adequate treatment of field singularities. Results presented show that good accuracy with moderate CPU time is obtained.

ACKNOWLEDGMENTS

Authors are grateful to Luis Ferragut and Eduardo Ballesteros for helpful discussions.

REFERENCES

(1) C.Tzuang, T.Itoh, "Finite-Element Analysis of Slow-Wave Schottky Contact Printed Lines". IEEE Trans. Microwave Theory Tech., vol.MTT-34, n.12, pp.1483-1489, Dec.1986.

(2) R.W.Lyon, J.Helszajn, "A Finite Element Analysis of Planar Circulators Using Arbitrarily Shaped Resonators". IEEE Trans. Microwave Theory Tech. vol.MTT-30, n.11, pp.1964-1974, Nov.1982.

(3) F.Hernández-Gil, J.Pérez-Martínez, "Analysis of Dielectric Resonators with Tuning Screw and Supporting Structure". IEEE Trans. Mic. Theory Tech., vol.MTT-33, n.12, pp.1453-1457, Dec.1985.

(4) I.V.Lindell, "On the Quasi-TEM Modes in Inhomogeneous Multiconductor Transmission Lines". IEEE Trans. Microwave Theory Tech., vol.MTT-29, n.8, pp.812-817, Aug.1981.

(5) R.Marqués, M.Horno, "Propagation of Quasi-Static Modes in Anisotropic Transmission Lines: Application to MIC Lines". IEEE Trans. Mic. Theory Tech., vol.MTT-33, n.10, pp.927-932, Oct.1985.

(6) Z.Pantic, R.Mittra, "Quasi-TEM Analysis of Microwave Transmission Lines by the Finite-Element Method". IEEE Trans. Microwave Theory Tech., vol.MTT-34, n.11, pp.1096-1103, Nov.1986.

(7) O.C.Zienkiewicz, "The Finite Element Method". McGraw-Hill, 1977.

(8) D.W.Kelly, J.P. de S.R.Gago, O.C.Zienkiewicz, I.Babuska, "A Posteriori Error Analysis and Adaptive Processes in the Finite Element Method: Part I-Error Analysis". Int.J.Num.Meth. Eng., vol.19, pp.1593-1619, 1983.

(9) J.W.Duncan, "The Accuracy of Finite-Differences Solutions of Laplace's Equation". IEEE Trans. Mic. Theory Tech., vol.MTT-15, n.10, pp.575-582, Oct.1967.

(10) J.Meixner, "The Behavior of Electromagnetic Fields at Edges". IEEE Trans.Antennas Propag., vol.AP-20, n.4, pp.442-446, Jul.1972.

(11) M.C.Rivara, "Algorithms for Refining Triangular Grids Suitable for Adaptive and Multigrid Techniques". Int. J. Num. Meth. Eng., vol.20, pp.745-756, 1984.

(12) P.Daly, "Upper and Lower Bounds to the Characteristic Impedance of Transmission Lines Using the Finite Element Method". COMPEL, Int.J.Comp. Math.Elec.Electronic Eng., vol.3, n.2, pp.65-78, 1984.

(13) C.T.Carson, G.K.Cambrell, "Upper and Lower Bounds on the Characteristic Impedance of TEM Mode Transmission-Lines". IEEE Trans.Mic.Theory Tech, vol.MTT-14, no.10, pp.497-498, Oct.1966.

(14) S.K.Koul, B.Bhat, "Propagation Parameters of Coupled Microstrip-Like Transmission Lines for Millimeter-Wave Applications". IEEE Trans.Mic. Theory Tech., vol.MTT-29, n.12, Dec.1981.